Interference Management in Cognitive Radio Systems — a Convex Optimisation Approach

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***Abstract*—We consider a cognitive radio system with N secondary user (SU) pairs and a pair of primary users (PU). The SU power allocation problem is formulated as a rate maximisation problem under PU and SU quality of service and SU peak power constraints. We show our problem formulation is a geometric program and can be solved with convex optimisation techniques. We examine the effect of PU transmissions in our formulations. Solutions for both low and high signal-to-interference-and-noise ratio (SINR) scenarios are provided. We show that including the PU rate in the optimisation problem leads to increased PU performance while not significantly degrading SU rate. Achievable rate cumulative distribution functions for various Rayleigh fading channels are produced.**

I. Introduction

A large number of papers have appeared on various aspects of cognitive radio (CR) systems, including fundamental information theoretic capacity limits (see, for example, [1–7]). In an underlay CR system the secondary users (SUs) protect the primary user (PU) by regulating their transmit power to maintain the PU receiver interference below a well defined threshold level. The limits on this received interference level at the PU receiver can be imposed by an average/peak constraint [2], or a minimum value for its signal-to-interference-and-noise ratio (SINR) [4]. While imposing an additional channel state information (CSI) requirement [5], the advantage of using an SINR-based PU protection mechanism is that it removes the constant interference threshold, thus benefiting the SUs when the PU link is strong.

Power control in conventional wireless networks has been extensively studied in the literature [8–10]. Power control in CR systems presents its own unique challenges. In spectrum sharing applications, SU power must be allocated in a manner that achieves the goals of the CR system while not adversely affecting the operation of the PU. Generally the goals of the CR are not compatible with the goals of the PU, for instance, increasing SU power to increase SU capacity will tend to increase interference to the PU. There is a growing body of literature on power control and capacity of CR systems. In [11], soft sensing information was used for optimal power control to maximise capacity of one SU pair coexisting with one PU pair. The impacts of SU transmission power on the occurrence of spectrum opportunities and the reliability of opportunity detection was analysed in [12]. In [13], dynamic programming was used to develop a power control strategy for one SU pair under a Markov model of the PU's spectrum usage. Optimal power allocation strategies to achieve the ergodic capacity and the outage capacity of one SU pair coexisting with one PU pair under different types of power constraints and fading channel models were obtained in [6]. Power control using game-theoretic approaches have been proposed in [14, 15]. Power control for CR systems using geometric programming have been proposed in [16–18]. In [17], a CR relay system with one cognitive source, one relay and a cognitive destination coexisting with a PU pair was considered and an optimisation problem to minimise the total CR transmit power under a peak interference constraint was formulated and solved using geometric programming. A minimax approach was used in [18] to minimise the maximum transmit power for a CR system coexisting with a PU-Rx. The interference caused by a PU-Tx to the SU-Rxs was not considered in the problem formulation of [18]. In [16], a distributed approach was used for power allocation to maximise SU sum capacity under a peak interference constraint, but the approach did not include the interference caused by the PU-Tx in the analysis and the problem was only analysed for a high SINR scenario.

Convex optimisation methods are widely used in the design and analysis of communications systems. Many problems that arise in communications signal processing can be cast or converted into convex optimisation problems which allow analytical or numerical solutions to be calculated easily [19]. In [20], several problems for designing optimal dynamic resource allocation in CR systems are formulated and the key role that convex optimisation plays in finding the optimal solutions is demonstrated.

In this paper we formulate the SU power allocation problem as a rate maximisation problem under PU and SU quality of service (QoS) and SU peak power constraints. We show that it can be solved using geometric programming and convex optimisation techniques. Unlike in [16–18], where the PU interference at each SU-Rx is neglected, we evaluate the effect of the PU interference by explicitly including it in our formulations. Solutions for both low and high SINR scenarios are presented. Most of the cognitive radio literature adopts a SU centric view and, apart from guaranteeing minimum QoS to PU, does not consider the PU-SU system as a whole. We demonstrate that considering the system rate in the optimisation problem results in improved PU performance without a significant penalty in SU rate. Rate cumulative distribution functions (CDFs) for various channel conditions are obtained

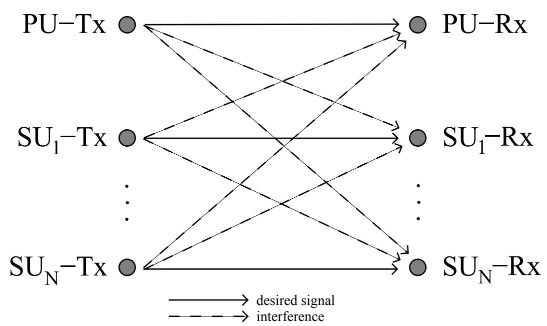

Fig. 1. System Model

through solution of our optimisation problems.

II. System Model

As shown in Fig. 1, we consider a cognitive radio system with a single PU and N SU transmitters communicating simultaneously over a common channel to their respective receivers. Independent, point-to-point, flat Rayleigh fading channels are assumed for all links in the network. Let $g_{\mathrm{p}} = |h_{\mathrm{p}}|^2$, $g_{\mathrm{ss}}^{(ij)} = |h_{\mathrm{ss}}^{(ij)}|^2$, $g_{\mathrm{ps}}^{(j)} = |h_{\mathrm{ps}}^{(j)}|^2$ and $g_{\mathrm{sp}}^{(i)} = |h_{\mathrm{sp}}^{(i)}|^2$ denote the instantaneous channel powers of the PU-Tx to PU-Rx, SU-Tx i to SU-Rx j, PU-Tx to SU-Rx j and SU-Tx i to PU-Rx links, respectively. For notational convenience we will denote $g_{\mathrm{s}}^{(i)} = g_{ss}^{ii}$. Furthermore, we assume that the channel powers for the PU and each of the N SUs are independent and identically distributed (iid) and are governed by their corresponding parameters $\Omega_{\mathrm{p}} = \mathbb{E}(g_p)$, $\Omega_{\mathrm{s}} = \mathbb{E}(g_{\mathrm{s}})$, $\Omega_{\mathrm{ss}} = \mathbb{E}(g_{\mathrm{ss}})$, $\Omega_{\mathrm{ps}} = \mathbb{E}(g_{\mathrm{ps}})$ and $\Omega_{\mathrm{sp}} = \mathbb{E}(g_{\mathrm{sp}})$. The $\mathbb{E}(\cdot)$ denotes the expectation operator.

In our model the SINR at the ith, $i = 1, \ldots, N$, SU receiver is given by

$$\gamma_{\mathrm{s}}^{(i)} = \frac{P_{\mathrm{s}}^{(i)} g_{\mathrm{s}}^{(i)}}{\sum_{j=1, j\neq i}^{N} P_{\mathrm{s}}^{(j)} g_{\mathrm{ss}}^{(ij)} + P_{\mathrm{p}} g_{\mathrm{ps}}^{(i)} + \sigma_{\mathrm{s}}^2} \tag{1}$$

and that at the PU receiver by

$$\gamma_{\mathrm{p}} = \frac{P_{\mathrm{p}} g_{\mathrm{p}}}{\sum_{i=1}^{N} P_{\mathrm{s}}^{(i)} g_{\mathrm{sp}}^{(i)} + \sigma_{\mathrm{p}}^2}, \tag{2}$$

where $P_{\mathrm{s}}^{(i)}$ and P_{p} are the ith SU and PU transmit powers, respectively, and σ_{s}^2 and σ_{p}^2 are the additive white Gaussian noise (AWGN) variance at the ith SU-Rx and PU-Rx, respectively. We also note that that there is a maximum transmit power constraint, $P_{\mathrm{s,max}}^{(i)}$, on the SU transmitters which may be due either to regulatory or hardware limitations. This is denoted by

$$P_{\mathrm{s}}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}.$$

Additionally, the non-negative vector $\mathbf{P}_{\mathrm{s}}$ is used to collectively refer to the set of SU transmit powers, i.e., $\mathbf{P}_{\mathrm{s}} \triangleq [P_{\mathrm{s}}^{(1)} \ldots P_{\mathrm{s}}^{(N)}]^T$.

In a cognitive radio system the secondary users are allowed to operate as long as they can guarantee a certain level of quality of service (QoS) to the primary user. Hence, in our analysis we impose an SINR constraint, γ_T, at the PU receiver

$$\gamma_{\mathrm{p}} \geq \gamma_{\mathrm{T}}.$$

The rate for a PU with a bandwidth of 1Hz is given by

$$R_{\mathrm{p}} = \log_2(1 + \gamma_{\mathrm{p}}), \tag{3}$$

while the SU sum rate is denoted by

$$R_{\Sigma} = \sum_{i=1}^{N} R_i, \tag{4}$$

where the individual rate of the ith SU with a bandwidth of 1Hz is given by

$$R_i = \log_2\left(1 + \gamma_{\mathrm{s}}^{(i)}\right). \tag{5}$$

Using (3) and (4), the system rate can then be expressed as

$$R_{\mathrm{sys}} = R_{\mathrm{p}} + R_{\Sigma}. \tag{6}$$

The main system variables can be parameterised as follows. We denote by

$$c_1 = \frac{\Omega_{\mathrm{sp}}}{\Omega_{\mathrm{s}}} \tag{7}$$

the ratio of interference to desired channel power. Similarly,

$$c_2 = \frac{\gamma_{\mathrm{T}}}{P_{\mathrm{p}}\Omega_{\mathrm{p}}/\sigma_{\mathrm{p}}^2} \tag{8}$$

represents the ratio of the minimum target SINR to the mean signal-to-noise ratio (SNR) at the PU-Rx. Hence, increasing c_2 corresponds to reducing the allowable interference, with the case of $c_2 = 1$ corresponding to zero average allowable interference. Finally,

$$c_3 = \frac{\Omega_{\mathrm{ss}}}{\Omega_{\mathrm{s}}} \tag{9}$$

parametrises the relative channel power of desired to interfering SU links.

III. SU Power Optimisation

In this section, we aim to find the SU power allocation such that the SU sum rate, R_{Σ}, or the system rate, R_{sys}, is maximised while maintaining the PU receiver QoS above the threshold γ_{T}, and keeping within the SU transmit power budget. We may optionally choose to set minimum SINR thresholds, $\gamma_{\mathrm{s,min}}^{(i)}$ on the ith SU receiver. This represents a practical limitation on SU receivers below which the receivers fail to operate with acceptable performance. In our formulation we assume that all channel gains are known which allows us to obtain fundamental limits on achievable rate. However, in practise the channel gains would need to be estimated, hence the rates obtained in this paper provide an upper bound. Mathematically we solve the following suite of optimisation problems.

1) SU Rate Maximisation with SU QoS Constraints:

$$\begin{aligned} \underset{\mathbf{P}_{\mathrm{s}}}{\text{maximise}} \quad & R_{\Sigma} \\ \text{subject to} \quad & \gamma_{\mathrm{p}} \geq \gamma_{\mathrm{T}} \\ & \gamma_{\mathrm{s}}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, \quad i = 1, \ldots, N \\ & P_{\mathrm{s}}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, \quad i = 1, \ldots, N \end{aligned} \tag{10}$$

2) SU Rate Maximisation without SU QoS Constraints:

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{maximise}} & R_\Sigma & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, \quad i = 1, \ldots, N & \end{array} \tag{11}$$

3) System Rate Maximisation with SU QoS Constraints:

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{maximise}} & R_\mathrm{sys} & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & \gamma_\mathrm{s}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, & i = 1, \ldots, N \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, & i = 1, \ldots, N \end{array} \tag{12}$$

4) System Rate Maximisation without SU QoS Constraints:

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{maximise}} & R_\mathrm{sys} & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, \quad i = 1, \ldots, N & \end{array} \tag{13}$$

From (4) and (5) it is obvious that maximising the objectives in (10)–(11) is equivalent to maximising

$$\prod_{i=1}^{N} \left(1 + \gamma_\mathrm{s}^{(i)}\right). \tag{14}$$

Similarly, for (12)–(13) we seek to maximise

$$(1 + \gamma_\mathrm{p}) \cdot \prod_{i=1}^{N} \left(1 + \gamma_\mathrm{s}^{(i)}\right). \tag{15}$$

Problems (10)–(13) can be modified to minimisation problems by taking the reciprocal of the objectives. The suite of optimisation problems are nonlinear and non-convex and generally hard to solve [19]. We proceed by dividing our problem into high and low SINR scenarios.

A. High SINR Scenario

When the SINR is sufficiently high, R_p, R_Σ and R_sys can be approximated by

$$\begin{aligned} R_\mathrm{p} &\approx \log_2(\gamma_\mathrm{p}) \\ R_\Sigma &\approx \log_2\left(\prod_{i=1}^{N} \gamma_\mathrm{s}^{(i)}\right) \\ R_\mathrm{sys} &\approx \log_2\left(\gamma_\mathrm{p} \cdot \prod_{i=1}^{N} \gamma_\mathrm{s}^{(i)}\right). \end{aligned} \tag{16}$$

Using the approximations in (16), the optimisation problems (10)–(13) can be written in minimisation form as

1) High SINR SU Rate Maximisation :

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{minimise}} & \prod_{i=1}^{N} \left(\frac{1}{\gamma_\mathrm{s}^{(i)}}\right) & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & \gamma_\mathrm{s}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, & i = 1, \ldots, N \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, & i = 1, \ldots, N \end{array} \tag{17}$$

2) High SINR System Rate Maximisation :

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{minimise}} & \left(\frac{1}{\gamma_\mathrm{p}}\right) \cdot \prod_{i=1}^{N} \left(\frac{1}{\gamma_\mathrm{s}^{(i)}}\right) & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & \gamma_\mathrm{s}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, & i = 1, \ldots, N \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, & i = 1, \ldots, N \end{array} \tag{18}$$

The second constraint in (17) and (18) is optional and only included if SU QoS constraints are required.

Problems (17) and (18) fall into a class of optimisation problems known as geometric programs (GP). A GP is stated as the following optimisation problem.

$$\begin{array}{lll} \text{minimise} & f_0(\mathbf{x}) & \\ \text{subject to} & f_i(\mathbf{x}) \leq 1, & i = 1, \ldots, m \\ & h_i(\mathbf{x}) = 1, & i = 1, \ldots, p, \end{array} \tag{19}$$

where $f_0, \ldots, f_m$ are in a form known as posynomials and $h_1, \ldots, h_p$ are referred to as monomials [19]. GPs are nonlinear, non-convex optimisation problems but can be transformed to convex optimisation problems by a logarithmic change of variables and by taking the logarithm of the objective and constraint functions [19]. The transformed problem can then be solved efficiently in polynomial time by interior point methods [21].

Through straightforward manipulation of the second and third constraints, problems (17) and (18) can be transformed into the standard form GP (19). Once in this form, they can be solved to obtain the optimum SU power allocation.

B. Low SINR Scenario

In the low SINR scenario our rate maximisation optimisation problems are given by

1) Low SINR SU Rate Maximisation :

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{minimise}} & \prod_{i=1}^{N} \left(\frac{1}{1 + \gamma_\mathrm{s}^{(i)}}\right) & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & \gamma_\mathrm{s}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, & i = 1, \ldots, N \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, & i = 1, \ldots, N \end{array} \tag{20}$$

2) Low SINR System Rate Maximisation :

$$\begin{array}{lll} \underset{\mathbf{P}_\mathrm{s}}{\text{minimise}} & \left(\frac{1}{1 + \gamma_\mathrm{p}}\right) \cdot \prod_{i=1}^{N} \left(\frac{1}{1 + \gamma_\mathrm{s}^{(i)}}\right) & \\ \text{subject to} & \gamma_\mathrm{p} \geq \gamma_\mathrm{T} & \\ & \gamma_\mathrm{s}^{(i)} \geq \gamma_{\mathrm{s,min}}^{(i)}, & i = 1, \ldots, N \\ & P_\mathrm{s}^{(i)} \leq P_{\mathrm{s,max}}^{(i)}, & i = 1, \ldots, N \end{array} \tag{21}$$

The second constraint in (20) and (21) is optional and only included if SU QoS constraints are required.

The objectives in problems (20) and (21) are ratios of posynomials and hence they are not themselves posynomials. Optimisation problems of this nature are not GP and are known as Complementary GP [22, 23]. Complementary GPs are non-convex problems but can be solved with an iterative technique known as the single condensation method [22, 23]. In each iteration, the feasible point computed in the previous iteration is used to approximate the denominator of the objective monomial. Since a ratio of posynomial and monomial is a posynomial, the resulting problem is a GP. The procedure is repeated until the solution converges to an optimum of the original Complementary GP. It should be noted that convergence to a local or global minimum is possible. The posynomial is approximated with a monomial using the geometric-arithmetic mean inequality

$$\sum_i \delta_i v_i \geq \prod_i v_i^{\delta_i} \tag{22}$$

where $v_i \geq 0$, $\delta_i \geq 0$ and $\sum_i \delta_i = 1$. If we let $u_i = \delta_i v_i$, then (22) can be written as

$$\sum_i u_i \geq \prod_i \left(\frac{u_i}{\delta_i}\right)^{\delta_i}. \tag{23}$$

Note that equality in (23) holds when $\delta_i = u_i / \sum_i u_i$. The term on the left hand side of (23) resembles the denominator of our objective, i.e. a sum of monomials. Hence, if we let $u_i(\mathbf{P}_\mathrm{s})$ be the monomial terms of the denominator and $\delta_i = u_i(\mathbf{P}_\mathrm{s}) / \sum_i u_i(\mathbf{P}_\mathrm{s})$, then from (23) it is clear that the denominator can be approximated around a feasible $\mathbf{P}_\mathrm{s}$ with a product of monomials. Since the approximation is always an under-estimator of the original posynomial, minimising the condensed objective guarantees that the solution moves towards a minimum of the original objective function.

For completeness, we present an algorithm that can be used for solving the low SINR rate maximisation problem [10, 22, 23]:

Algorithm 1 Single Condensation Method

1. Generate a random feasible vector $\tilde{\mathbf{P}}_\mathrm{s}$.
2. Compute the individual monomial terms, $u_i(\tilde{\mathbf{P}}_\mathrm{s})$, and the denominator, $\sum_i u_i(\tilde{\mathbf{P}}_\mathrm{s})$, of the objective function using $\tilde{\mathbf{P}}_\mathrm{s}$.
3. Using results from step 2, compute δ_i with $\delta_i = u_i(\tilde{\mathbf{P}}_\mathrm{s}) / \sum_i u_i(\tilde{\mathbf{P}}_\mathrm{s})$.
4. Using δ_i, form the condensed denominator, $\prod_i \left(u_i(\mathbf{P}_\mathrm{s})/\delta_i\right)^{\delta_i}$. Note $\mathbf{P}_\mathrm{s}$ is the optimisation variable.
5. Solve the resulting GP and assign solution to $\tilde{\mathbf{P}}_\mathrm{s}^l$, where l is the loop iteration.
6. Exit loop if $\|\tilde{\mathbf{P}}_\mathrm{s}^l - \tilde{\mathbf{P}}_\mathrm{s}^{l-1}\| \leq \epsilon$, where ϵ is the error tolerance.
7. GOTO step 2 with $\mathbf{P}_\mathrm{s}^l$ computed in step 5.

IV. Simulation Results and Discussion

We now present simulation results of the optimisation problems formulated in Section III, specifically evaluating the CDFs of the resulting rates. We consider a system with $N = 3$ SUs. In all simulations we have set $P_\mathrm{p}/\sigma_\mathrm{p}^2 = 0$ dB and $\Omega_\mathrm{p}/\sigma_\mathrm{p}^2 = \Omega_\mathrm{s}/\sigma_\mathrm{s}^2 = 6.5$ dB, where we assume $\sigma_\mathrm{p}^2 = \sigma_\mathrm{s}^2$. Simulations for problems (10) and (12) have $\gamma_{\mathrm{s,min}}^{(i)} = -10$ dB, $i = 1, \ldots, N$. The optimisation problems are solved using the CVX solver [24]. We consider the following three channel scenarios

1) *Scenario A*: Low Interference
   In this scenario $c_1 = c_3 = 0.1$ which corresponds to each receiver being approximately 3 times (assuming $1/d^2$ path loss) further away from the interfering transmitters than its own transmitter. This results in low interference between all users, thus making the PU QoS constraint easy to satisfy.
2) *Scenario B*: High Interference
   In this scenario $c_1 = c_3 = 0.9$ which corresponds to each receiver being approximately the same distance from all transmitters. This results in high interference among all users, thus making the PU QoS constraint difficult to satisfy.
3) *Scenario C*: Low PU and High SU Interference
   In this scenario $c_1 = 0.1$ and $c_3 = 0.9$. Here the PU experiences low interference from the SUs since it is approximately 3 times further away from SU-Txs than the PU-Tx. As a result, the PU QoS constraint is easily satisfied. However, SU to SU interference is very prominent.

Results of our proposed methods are compared against the equal power allocation method and a power profile method analogous to the "poor man's waterfilling" method [25] where we allocate power proportionally to $g_\mathrm{s}^{(i)}/g_\mathrm{sp}^{(i)}$. We refer to these methods as *ad hoc* allocation methods. Note that the *ad hoc* allocation methods do not impose a minimum SU QoS requirement, hence a fair comparison is only possible against problems (11) and (13). Figures 2–6 show SU sum and PU rate CDF obtained from optimisation problems (10)–(13) for the three channel conditions with $\gamma_\mathrm{T} = 2$ dB.

Figure 2 shows the SU sum rate CDF of *Scenario A* along with results of *ad hoc* allocation methods. We observe that problems (10) and (12) result in almost the same performance. Due to PU and SU QoS requirements, we observe that around 50% of the time no SUs are able to access the channel. Similarly, problems (11) and (13) result in very similar performance and, due to the PU QoS requirement, no SUs are able to transmit around 30% of the time. Furthermore, we see that the *ad hoc* allocation methods are outperformed by the GP methods. Figure 3 shows the PU rate CDF resulting from the four optimisation problems along with the CDF for the reference case when no SUs are transmitting. It is seen that the SU power allocation has minor effect on PU operation due to favourable values of system parameters c_1 and c_3.

Figure 4 shows the SU sum rate CDF of *Scenario B* along with results of *ad hoc* allocation methods. Once again, we observe that problems (10) and (12) result in almost the same performance. Due to PU and SU QoS requirements, around 80% of the time no SUs are able to access the channel. A solution to this difficulty has been derived, and will be presented in a future paper. Problems (11) and (13) result in somewhat similar performance and, due to the PU QoS

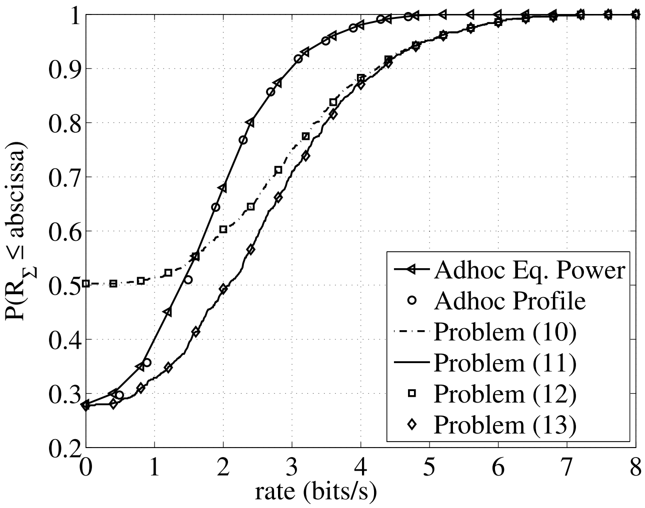

Fig. 2. R_Σ CDF for *Scenario A*, $\gamma_\mathrm{T} = 2$ dB.

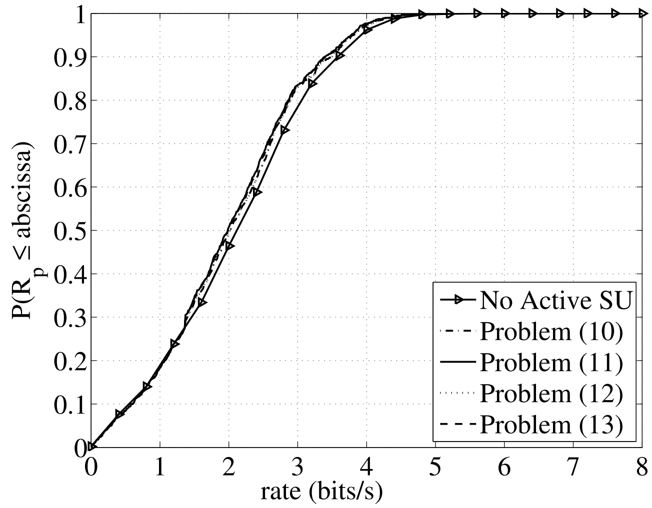

Fig. 3. R_p CDF for *Scenario A*, $\gamma_\mathrm{T} = 2$ dB.

requirement, no SUs are able to transmit 47% of the time. Figure 5 shows the PU rate CDF and the effect of SUs transmission. The discontinuity in the graph corresponds to the point where the optimisation problems become feasible and SU transmissions start. Compared to problem (11), we see that maximising the system rate, problem (13), results in improved PU performance while not significantly affecting the SU performance. This implies that it pays to consider the system rate rather than just SU sum rate.

Figure 6 shows the SU sum rate CDF of *Scenario C* along with results of *ad hoc* allocation methods. Problems (10) and (12) result in the same performance. Due to PU and SU QoS requirements around 57% of the time no SUs are able to access the channel. Similarly, problems (11) and (13) result in same performance and due to the PU QoS requirement no SUs are able to transmit around 30% of the time. As expected, higher rates are achieved in *Scenario A* compared to *Scenario C*. Figure 7 shows PU rate and, as for *Scenario A*, SU power allocation has minor effect on the PU.

The mean SU sum rate—problems (11) and (13)—is plotted in Figure 8 as a function of γ_T for *Scenarios A–C*. We observe that the performance for problems (11) and (13) is very similar and this reaffirms our finding that considering the system rate in the optimisation problem does not significantly degrade the SU performance.

V. Conclusions

In this paper, we have formulated the SU power allocation problem in a CR system as a geometric program and obtained achievable rate CDFs in various channel conditions. We have included the effect of PU transmission in our formulations and studied the problem in both high and low SINR scenarios. More importantly, we have shown that considering system rate optimisation improves the PU performance while not significantly degrading the SU performance.

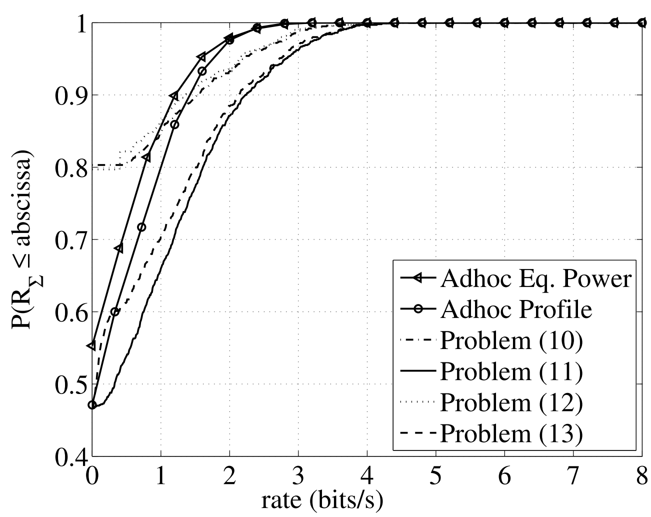

Fig. 4. R_Σ CDF for *Scenario B*, $\gamma_\mathrm{T} = 2$ dB.

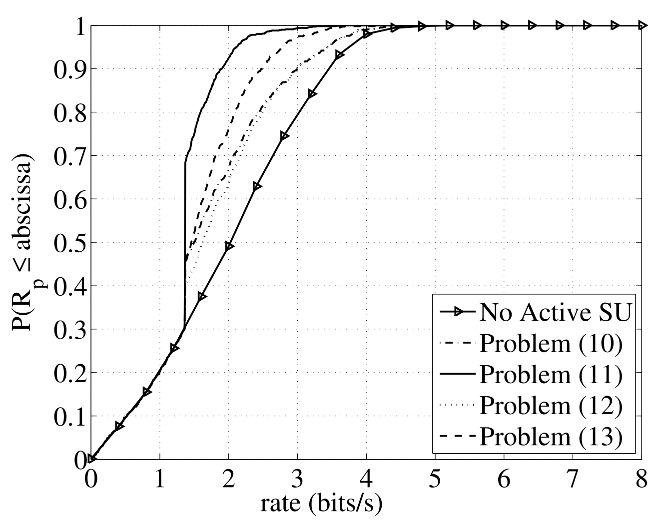

Fig. 5. R_p CDF for *Scenario B*, $\gamma_\mathrm{T} = 2$ dB.

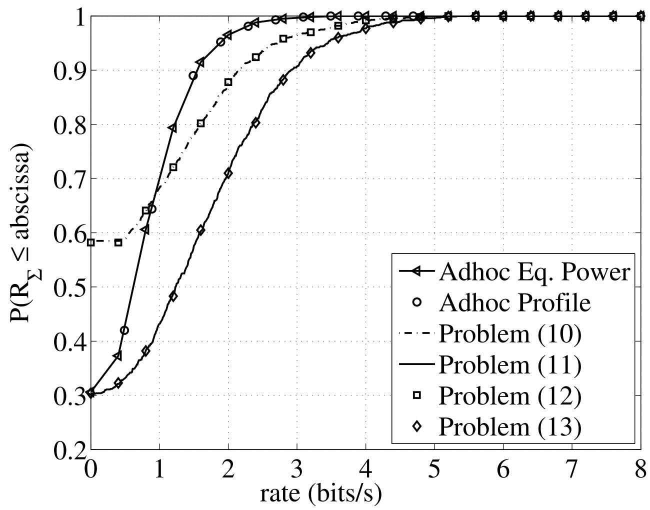

Fig. 6. R_Σ CDF for *Scenario C*, $\gamma_{\mathrm{T}} = 2$ dB.

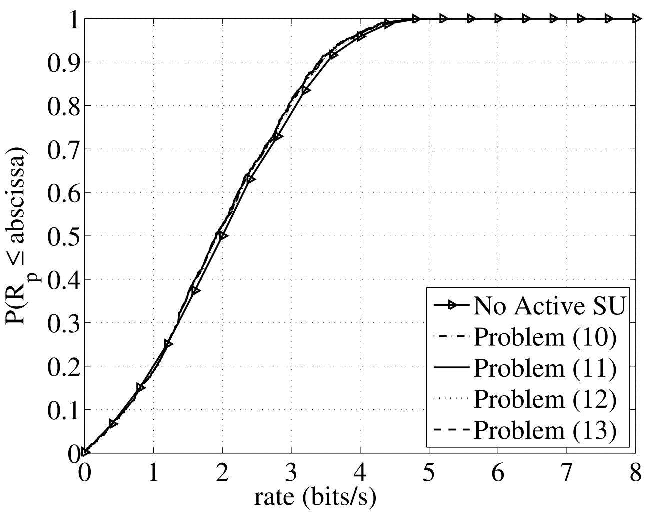

Fig. 7. R_{P} CDF for *Scenario C*, $\gamma_{\mathrm{T}} = 2$ dB.

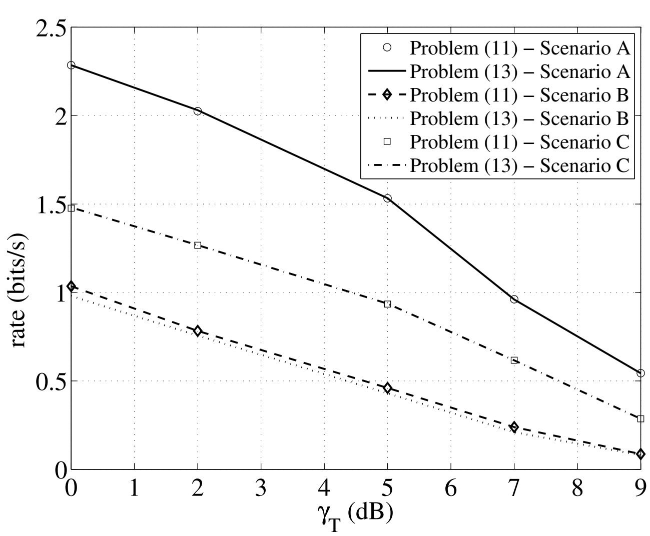

Fig. 8. Mean R_Σ as a function of γ_{T}.

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